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THE PROBLEM OF THE STABILITY OF VIBRATIONS OF A DISCRETE MECHANICAL SYSTEM PROTECTED FROM VIBRATIONS

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Annotation: In this work, the issue of analytically verifying the priority of vibrations of a mechanical system with elastic dissipative characteristics of the hysteresis type under the influence of random parametric excitations together with a dynamic damper is considered, depending on the design parameters of the system. In this case, solutions are sought in the form of exponents, a system of equations in normal form is formed, and the system of motion differential equations is averaged using the averaging method. The system of motion differential equations of the vibration-protected system is brought to the system of Ito differential equations. Based on the solutions obtained in the form of exponents, characteristic equations and their roots are determined. According to the main theorem of priority theory, analytical expressions of priority conditions are obtained based on the roots of characteristic equations. The boundaries of the priority and unstable regions are determined and analyzed in an analytical form.
Key words: mechanical system, dynamic damper, hysteresis, dissipative, random parametric excitation, stochastic process, root mean square value, priority conditions**.**

1 INTRODUCTION

There are many scientific studies on the priority of motion in mechanical systems. These studies analyze the priority of motion for various processes and mechanical systems and provide necessary recommendations for ensuring priority.

In the work [1], the question of stability of mechanical systems with hysteresis-type connections in random excitations was studied. In this case, the differential equations of motion of the systems were reduced to the Ito differential equation using the stochastic averaging method, and the stability of motion was checked using the Lyapunov method. In order to check the reliability of stability conditions, the issues were solved and analyzed.

[2] - the work evaluates the potential of a dynamic damper with nonlinear characteristics in absorbing the energy of parametric vibrations in a mechanical system. It is shown that its efficiency in absorbing the energy of parametric vibrations is high when the frequency is tuned to the resonance frequency. Recommendations are given to increase the efficiency of the dynamic damper in parametric excitations. In addition, information is provided about the basic principles of parametric vibrations, their advantages and disadvantages in damping.

[3] - in the work, several linearization methods are proposed for nonlinear mechanical systems with parametric white noise excitations. Differential equations are obtained to determine the root mean square values of linear mechanical systems. It is shown that various nonlinear models can be reduced to linearization coefficients when linearizing

[4] - the work presents methods for using stochastic methods in solving problems of protecting various mechanical systems from harmful vibrations in random processes, and develops recommendations.

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The work [5] considered the issues of stability of longitudinal and transverse vibrations of the rod and beam under the influence of external forces. In this case, a random function was selected and the values at which stability conditions were determined through the spectral density of longitudinal and transverse vibrations in white noise excitations.

In the work [6], the mean square value and stability in random parametric excitations of mechanical systems were studied. Ito's differential equation is derived by the stochastic averaging method, and it is shown that the obtained results are also appropriate for the stability of system motion. System motion stability conditions are expressed depending on the spectral density of vibrations. Changes of stability and unstability fields depending on the damping coefficient were numerically analyzed.

[7] - the work considered the vibrations of a class of mechanical systems under the influence of parametric excitations. In this case, parametric excitations were considered multi-frequency. Taking into account the nonlinear dissipative property of materials of mechanical systems, equations of motion were derived. In expressing the nonlinearity and damping force in materials, the second and third order nonlinear terms were limited. Van der Pol and Rayleigh equations were presented as special cases. Resonance states at several frequencies of parametric excitation were analyzed. Recommendations were given on adjusting frequencies to ensure optimal motion

[8] - the work systematically describes the basic principles of the theory of random functions used in various practical areas. Much attention is paid to the correlation theory of random processes and the determination of probabilistic properties of dynamical systems. Along with systems represented by ordinary differential equations, systems described by partial differential equations (distributed parameter systems) are also studied. The problem of determining the transfer function of a linear system that minimizes the error variance for given characteristics of the useful signal and noise is discussed.

[9]- the article studies nonlinear parametric vibrations of a rod with a dynamic damper under the influence of external excitations, taking into account the elasticity and damping properties of materials. The linearization method is used to solve nonlinear differential equations of motion of the system. The non-stationary and stationary values of the amplitude and phase of the vibrations are determined analytically. The priority conditions of stationary motion are obtained based on the Rous-Hurwitz criterion. The effect of changing the parameter values on the amplitude-frequency characteristic constructed based on the calculation results is shown.

In the article [10], the problem of studying nonlinear parametric vibrations of a rod combined with an element with frictional properties under the influence of random excitations is solved. The nonlinearity is taken in the form of a cubic degree polynomial of the Winkler type. The differential equation representing the vibrations of the rod is determined, it is shown that it does not have an exact analytical solution, and an approximate solution is proposed based on the linearization method. The proposed solution is compared with the proposed Monte Carlo solution based on a numerical approach and the results are shown to be in good agreement.

[11-14] - in the works, methods have been developed for mathematical modeling of nonlinear mechanical systems protected from vibrations, studying their dynamics and checking the priority of their movements, as well as determining the parameters corresponding to the priority movements, and evaluating the influence of the characteristics of the damping properties of the materials of the mechanical system. The expressions of modal mass and modal stiffness are expressed analytically. With their help, the selection and modeling of the materials of the systems are justified.

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The issue of verifying the priority of nonlinear vibrations of a mechanical system with hysteresis-type elastic dissipative characteristics under the influence of random parametric excitations in combination with a dynamic damper is one of the urgent problems that require solution.

2 Materials and method

This work deals with the issue of checking the priority of nonlinear vibrations of a mechanical system with elastic dissipative characteristics of the hysteresis type under the influence of random parametric excitations, together with a dynamic damper. The dissipative properties of the mechanical system and the dynamic damper materials are obtained in the hysteresis type. The relationships between stress and deformation are obtained in terms of nonlinear single-valued functions and expressed in linear functions using the statistical linearization method.

We can express the system of differential equations of motion of a mechanical system protected from vibrations as follows:

$$\ddot{x}_1 + \omega_1^2(1-\eta_1+i\eta_2)x_1 - \mu\omega_2^2(1-\nu_1+i\nu_2)x_2 = -\omega_1^2\xi_0(t)x_1(t);$$

(1)

$$\ddot{x}_1 + \ddot{x}_2 + \omega_2^2(1-\nu_1+i\nu_2)x_2 = 0,$$

where x_1, x_2 – are the displacement coordinates of the protected object and the dynamic absorber; $\omega_1^2 = \frac{c_1}{m_1}$ - natural frequency of the object to be protected from vibrations; c_1 and m_1 - are the stiffness and mass of the object being protected from vibrations; $\omega_2^2 = \frac{c_2}{m_2}$ -the eigen frequency of the dynamic damper;c_2 and m_2 -are the stiffness and mass of the dynamic damper; $i^2 = -1$; η_1, η_2 and ν_1, ν_2 - are linearization coefficients expressing the dissipative properties of the material of the elastic damping elements of the object protected from vibrations and the dynamic damper, respectively; $\mu = \frac{m_2}{m_1}$; $\xi_0(t)$ - a variable dimensionless quantity representing a stationary normal random process.

We search for solutions of the system of differential equations (1) in the form of exponents and take into account that in stochastic processes the variables of the system of differential equations of motion satisfy the Ito equations. As a result, using the stochastic averaging method, we reduce the system of differential equations of motion (1) to the system of Ito differential equations. If we determine the characteristic equation of the system of Ito differential equations, its roots will be as follows:

$$\lambda_1 = \frac{1}{2}\left(p_1 + \frac{\pi}{2}p_5^2\big(S(0) + i\psi(2\omega)\big) + p_3\right) + \frac{1}{8\omega^2}(\alpha_1 + i\beta_1);$$

$$\lambda_2 = \frac{1}{2}\left(p_1 + \frac{\pi}{2}p_5^2\big(S(0) + i\psi(2\omega)\big) + p_3\right) - \frac{1}{8\omega^2}(\alpha_1 + i\beta_1);$$

(2)

$$\lambda_3 = \frac{1}{2}\left(-p_1 - \frac{\pi}{2}p_5^2\big(S(2\omega) - S(0)\big) - p_3\right) + \frac{1}{8\omega^2}(\alpha_2 + i\beta_2);$$

$$\lambda_4 = \frac{1}{2}\left(-p_1 - \frac{\pi}{2}p_5^2\big(S(2\omega) - S(0)\big) - p_3\right) - \frac{1}{8\omega^2}(\alpha_2 + i\beta_2),$$

where

$$p_1 = \frac{\omega^2 - \omega_1^2(1-\eta_1+i\eta_2)}{2i\omega}; \quad p_3 = \frac{\omega^2 + (1+\mu)\omega_2^2(1-\nu_1+i\nu_2)}{2i\omega};$$

$$p_5 = -\frac{\omega_1^2}{2i\omega};$$

and $S(0),\ S(2\omega),\ \psi(2\omega)$are defined as the spectral density of a stationary normal random process $\xi_0(t)$ as follows [4]:

$$S(2\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}R(\tau)cos\omega\tau d\tau;\ \ \psi(2\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}R(\tau)sin\omega\tau d\tau,$$

$R(\tau)=E[\xi_{0n}(t)\xi_{0m}(t+\tau)]=\langle\xi_{0n}(t)\xi_{0m}(t+\tau)\rangle$ - is correlation function; $\omega -$ frequency of vibrations;

$$\alpha_1=2^{-\frac{1}{2}}(a_1^2+b_1^2)^{\frac{1}{4}}((a_1^2+b_1^2)^{\frac{1}{2}}+a_1)^{\frac{1}{2}};$$

$$\alpha_2=2^{-\frac{1}{2}}(a_2^2+b_2^2)^{\frac{1}{4}}((a_2^2+b_2^2)^{\frac{1}{2}}+a_2)^{\frac{1}{2}};$$

$$\beta_1=2^{-\frac{1}{2}}(a_1^2+b_1^2)^{\frac{1}{4}}((a_1^2+b_1^2)^{\frac{1}{2}}-b_1)^{\frac{1}{2}};$$

$$\beta_2=2^{-\frac{1}{2}}(a_2^2+b_2^2)^{\frac{1}{4}}((a_2^2+b_2^2)^{\frac{1}{2}}-b_2)^{\frac{1}{2}};$$

$$a_1=16((\eta_2^2-(1-\eta_1)^2)\omega_1^4-6((1-\eta_1)(1-\nu_1)+\eta_2\nu_2)\left(\mu+\frac{1}{3}\right)\omega_2^2\omega_1^2-$$
$$-(1+\mu)^2((1-\nu_1)^2-\nu_2^2)\omega_2^4)\omega^2+8\omega_1^4\pi((1-\eta_1)\omega_1^2-(1-\nu_1)\omega_2^2(\mu-$$
$$-1))\psi(2\omega)+\left(\eta_2\omega_1^2-\nu_2\omega_2^2(\mu-1)\right)S(0))\omega-\left(\left(\psi(2\omega)\right)^2-S(0)^2\right)\pi^2\omega_1^8;$$

$$b_1=-32((1-\eta_1)\eta_2\omega_1^4+3\left(\eta_2(1-\nu_1)+\nu_2(1-\eta_1)\right)\left(\mu+\frac{1}{3}\right)\omega_2^2\omega_1^2+$$
$$+(1+\mu)^2\omega_2^4(1-\nu_1)\nu_2)\omega^2+8\omega_1^4\pi(\left(\eta_2\omega_1^2-\nu_2\omega_2^2(\mu-1)\right)\psi(2\omega)+$$
$$+\left(-(1-\eta_1)\omega_1^2+(1-\nu_1)\omega_2^2(\mu-1)\right)s(0))\omega+2\omega_1^8\pi^2S(0)\psi(2\omega);$$

$$a_2=16((\eta_2^2-(1-\eta_1)^2)\omega_1^4-6((1-\eta_1)(1-\nu_1)-\eta_2\nu_2)\omega_2^2\left(\mu+\frac{1}{3}\right)\omega_1^2-$$
$$-(1+\mu)^2\omega_2^4((1-\nu_1)^2-\nu_2^2))\omega^2+8\omega_1^4\pi\left(\eta_2\omega_1^2-\nu_2\omega_2^2(\mu-1)\right)\times$$
$$\times(S(2\omega)-S(0))\omega+\pi^2\omega_1^8(S(0)-S(2\omega))^2;$$

$$b_2=-32((1-\eta_1)\eta_2\omega_1^4+3\left(\eta_2(1-\nu_1)+\nu_2(1-\eta_1)\right)\left(\mu+\frac{1}{3}\right)\omega_2^2\omega_1^2+$$
$$+(1+\mu)^2\omega_2^4\times\omega_2^4(1-\nu_1)\nu_2)\omega^2+8\omega_1^4\pi(-(1-\eta_1)\omega_1^2+(1-\nu_1)\times$$
$$\times\omega_2^2(\mu-1))\left(S(2\omega)-S(0)\right)\omega.$$

Let us consider the case where the expressions$p_1,\ p_3,p_5$ in the roots of the characteristic equation (2) are real. According to the priority theory, for the considered action to be asymptotically priority, it is sufficient that the real part of the roots of the characteristic equation be negative [4]. After some substitutions, we obtain the following priority conditions:

$$-\omega_1^2\eta_2-\pi\frac{\omega_1^4}{4\omega}S(0)+(1+\mu)\omega_2^2\nu_2>0;$$

(3)

$$\omega_1^2\eta_2+\pi\frac{\omega_1^4}{4\omega}\left(S(2\omega)-S(0)\right)-(1+\mu)\omega_2^2\nu_2>0.$$

The resulting system of inequalities (3) is the priority condition for the oscillations of a mechanical system with elastic dissipative characteristics of the hysteresis type under the influence of random parametric excitations, together with a dynamic damper.

3 Results and discussion

We analyze the obtained priority conditions (3), for which we obtain the expression of the spectral density $S(\omega)$ in the following form [8]:

$$S(\omega) = \frac{\sigma_\xi^2}{\pi} \cdot \frac{q}{\omega^2 + q^2}, \tag{4}$$

where $\sigma_\xi -$ is the mean square value of the base excitation; $q -$ is dominant frequency.

(4) We define the correlation function based on the spectral density expression. For this, we use the following relationship [4]:

$R(\tau) = 2\int_0^\infty S(\omega) cos\omega\tau d\omega.$ (5)

When calculating the correlation function (5), we take into account that the trigonometric function $cos\omega\tau = (e^{i\omega\tau} + e^{-i\omega\tau})/2$. Then, if we substitute the spectral density expression (4) into the correlation function (5), we get

$$R(\tau) = 2\int_0^\infty \frac{\sigma_\xi^2}{\pi} \cdot \frac{q}{\omega^2 + q^2} \frac{e^{i\omega\tau} + e^{-i\omega\tau}}{2} d\omega. \tag{6}$$

We replace the expression $\frac{q}{\omega^2+q^2}$ in the correlation function expression (6) with the following approximate expression:

$$\frac{q}{\omega^2 + q^2} \approx \frac{1}{q} e^{-\frac{\omega^2}{q^2}}. \tag{7}$$

(7) The approximate substitution has sufficient accuracy, as we can see the graphs of these two functions below:

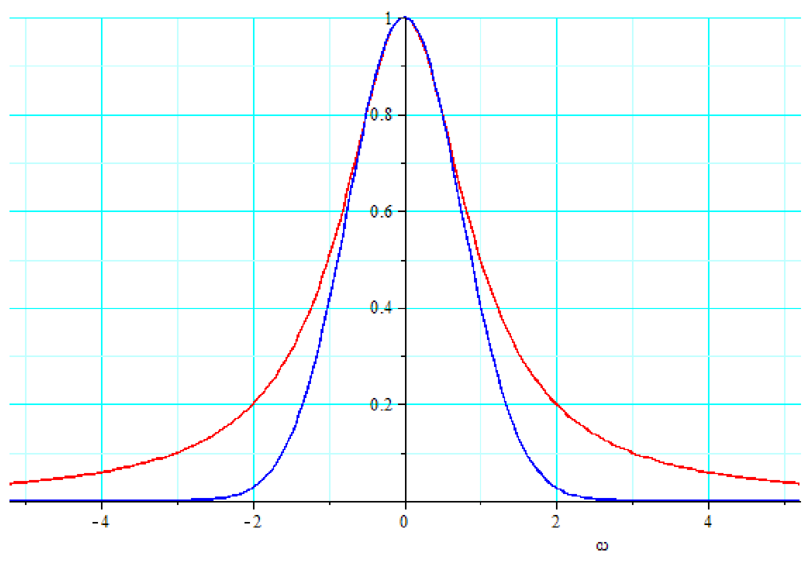

Figure 1. $\frac{q}{\omega^2+q^2}$ *and* $\frac{1}{q}e^{-\frac{\omega^2}{q^2}}$ *graph of functions (red, blue), (q=1).*

Taking into account the substitution (7), $R(\tau)$ becomes:

$$R(\tau) = \frac{\sigma_\xi^2}{\pi q} \int_0^\infty (e^{i\omega\tau - \frac{\omega^2}{q^2}} + e^{-i\omega\tau - \frac{\omega^2}{q^2}}) d\omega = \sigma_\xi^2 e^{-q|\tau|}. \tag{8}$$

According to the defined correlation function (8), we define the spectral density expression $\psi(2\omega)$

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$$\psi(2\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R(\tau)sin\omega\tau d\tau = \frac{\sigma_\xi^2}{2\pi}\int_{-\infty}^{\infty} e^{-q|\tau|}sin\omega\tau d\tau. \quad (9)$$

When calculating the spectral density expression (9), we take into account that the trigonometric function in it is $sin\omega\tau = (e^{i\omega\tau} - e^{-i\omega\tau})/2i$. In this case, the value of the spectral density expression (9) will be as follows:

$$\psi(2\omega) = \frac{\sigma_\xi^2}{2\pi}\int_{-\infty}^{\infty} e^{-q|\tau|}\cdot\frac{e^{i\omega\tau}-e^{-i\omega\tau}}{2i}d\tau = \frac{\sigma_\xi^2}{4\pi i}\int_{-\infty}^{\infty}\left(e^{i\omega\tau-q|\tau|} - e^{-i\omega\tau-q|\tau|}\right)d\tau =$$

$$= \frac{\sigma_\xi^2}{4\pi i}\left[\int_{-\infty}^{0}\left(e^{(i\omega+q)\tau} - e^{(-i\omega+q)\tau}\right)d\tau + \int_{0}^{\infty}\left(e^{(i\omega-q)\tau} - e^{(-i\omega-q)\tau}\right)d\tau\right] =$$

$$= \frac{\sigma_\xi^2}{4\pi i}\left[\left(\frac{e^{(i\omega+q)\tau}}{i\omega+q} - \frac{e^{(-i\omega+q)\tau}}{-i\omega+q}\right)\Big|_{-\infty}^{0} + \left(\frac{e^{(i\omega-q)\tau}}{i\omega-q} - \frac{e^{(-i\omega-q)\tau}}{-i\omega-q}\right)\Big|_{0}^{\infty}\right] = 0. \quad (10)$$

Considering the expressions for spectral densities (4) and (10), the priority conditions (3) are as follows:

$$\frac{1}{\omega^2 q}\left(\omega_1^4\sigma_\xi^2 - 4\omega q\left((1+\mu)\nu_2\omega_2^2 - \eta_2\omega_1^2\right)\right) < 0;$$

(11)

$$\frac{\left((1+\mu)\nu_2\omega_2^2 - \eta_2\omega_1^2\right)(q^3 + 4\omega^2 q) + \omega_1^4\sigma_\xi^2\omega}{(4\omega^2+q^2)\omega q} < 0.$$

The case where $\omega = \omega_1$ is of practical importance, that is, it allows analyzing the oscillations of the system around the resonant frequency. When $\omega_1 = \omega_2$ and $\omega = q$, the priority conditions (11) are as follows:

$$\sigma_\xi^2 - 4\left((1+\mu)\nu_2 - \eta_2\right) < 0;$$

(12)

$$\sigma_\xi^2 + 5\left((1+\mu)\nu_2 - \eta_2\right) < 0.$$

From the obtained priority conditions (12), it can be seen that for the considered case, the motion of the vibration-protected system is unstable. However, if $(1+\mu)\nu_2 > \eta_2$, the oscillations of the accumulated mass in the system take precedence, and the oscillations of the dynamic damper are unstable. When $(1+\mu)\nu_2 < \eta_2$, the opposite is true, that is, the oscillations of the accumulated mass are unstable, and the oscillations of the dynamic damper are dominant.

We analyze the change in the priority domain and boundary.For the case where $\sigma_\xi = \sqrt{10}g; \eta_1 = 1.196826491\eta\sigma_{x_1}; \eta_2 = 0.7978843277\eta\sigma_{x_1}; \nu_1 = 0;$ $\nu_2 = 0;\ \Omega_0 = \frac{\omega}{q}$ we draw graphs of functions expressing the priority conditions (11):

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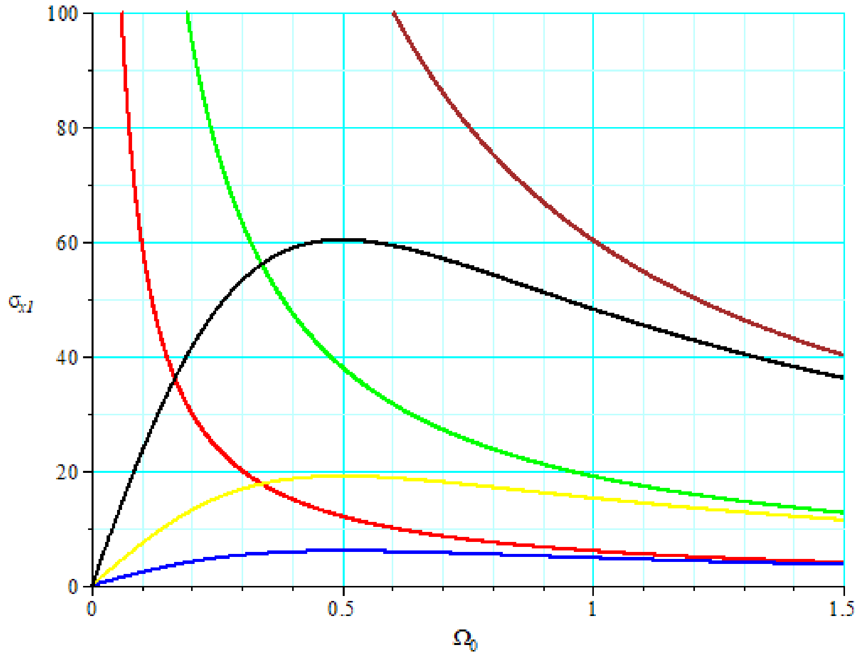

Figure 2. Change in the priority domain defined by expressions (11).

Figure 2 shows the variation of the functions representing the lower and upper boundaries of the priority domain defined by expressions (11) depending on the dimensionless parameter Ω_0 for different values of the parameter η, which represents the dissipative properties of the material (in the figure, the value of σ_{x_1} is magnified by 10^4 times). When $\eta = 5 \cdot 10^5; 5 \cdot 10^6; 5 \cdot 10^7 m^{-2}$, it can be seen from the graphs of the functions representing the lower (red, green, brown) and upper (blue, yellow, black) boundaries of the priority domain that in all three cases the lower and upper boundaries do not intersect and form a closed domain. This indicates the instability of the considered motion at the selected values of the parameters. With an increase in the dimensionless parameter Ω_0, the lower and upper boundaries approach each other but do not intersect, that is, they do not form a priority domain.

4 Conclusion

A method for checking the priority of oscillations of a discrete mechanical system with elastic dissipative characteristics of the hysteresis type, protected from oscillations under the influence of random parametric excitations, was developed using the Ito method. The priority conditions of the system were determined analytically for different values of the parameters. The expressions of the determined priority conditions allow us to select system parameters corresponding to priority actions in different forms of spectral densities of random processes.

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