ISSN: 2582-4686 SJIF 2021-3.261,SJIF 2022-2.889, 2024-6.875 ResearchBib IF: 10.25 / 2025

THE MULTIDISCIPLINARY JOURNAL OF SCIENCE AND TECHNOLOGY

VOLUME-6, ISSUE-3

ENG MASHHUR TEOREMA

Termiz davlat universiteti akademik litseyi matematika fani oqituvchisi

Karimova Xalima Samatovna

Termiz davlat universiteti akademik litseyi fizika fani oqituvchisi

Xidirova Maftuna Jovli qizi

Annotatsiya

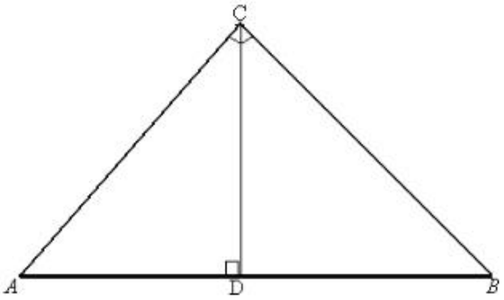
Ushbu maqolada umumiy o‘rta ta‘lim maktablarining yuqori sinflari,akademik litsey va professional ta ‘lim Geometriya kursida uchraydigan uchburchaklar va to‘rtburchaklarda uchraydigan masalalari va ularni yechimlarikeltirilgan

Kalit so‘zlar:*Pifagor,katet,gipotenuza,o’tkir burchak, perpendikulyar, og’ma,*

Yunon olimi Pifagor to`g`ri burchakli uchburchak tomonlari orasidagi mavjud bo`lgan munosabatlarni aniqlab, uni quyidagicha isbotlaydi.

<u>Teorema (Pifagor teoremasi)</u>. To`g`ri burchakli uchburchak gipotenuzasining kvadrati katetlar kvadratlarining yiqindisiga teng.

Isboti. *ABC* berilgan to`g`ri burchakli uchburchak bo`lib, unda burchak $C= 90^0$ – to`g`ri burchak bo`lsin. To`g`ri burchakning *C* uchidan *CD* balandlikni o`tkazamiz (1- rasm).



1-rasm.

Burchak kosinusining ta’rifiga ko`ra to`g`ri burchakli uchburchaklar *ABC* va *ACD* ning o`tkir burchagi *A* uchun:

$$\cos\angle A = \frac{AD}{AC} = \frac{AC}{AB}$$, bundan , $AB \cdot AD = AC^2$.

shunga o`xshash

$$\cos\angle B = \frac{BD}{BC} = \frac{BC}{AB}$$, bundan . $AB \cdot BD = BC^2$.

ISSN: 2582-4686 SJIF 2021-3.261,SJIF 2022-2.889, 2024-6.875 ResearchBib IF: 10.25 / 2025

THE MULTIDISCIPLINARY JOURNAL OF SCIENCE AND TECHNOLOGY

VOLUME-6, ISSUE-3

Hosil bo`lgan tengliklarni hadma-had qo`shib va $AD ++ DB= AB$ ekanini hisobga olib, $AC^2 + +BC^2 = AB(AD + +DB)= AB^2$ tenglikni hosil qilamiz. Teorema isbotlandi.

Pifagor teoremasidan ushbu natija kelib chiqadi:

To`g`ri burchakli uchburchakning istalgan kateti gipotenuzasidan kichik.

Bundan o`z navbatida quyidagi natija kelib chiqadi:

har qanday o`tkir burchak uchun cos<1.

Mashqlar

1. To`g`ri burchakli uchburchakning a va b katetlari berilgan. Gipotenuzani toping:

1) $a= 3$; $b= 4$; 2) $a=1$; $b= 1$; 3) $a= 5$; $b= 6$.

(*Javob*: 1) 5; 2) $\sqrt{2} \approx 1{,}1$; 3) $\sqrt{61} \approx 7{,}8$).

2. To`g`ri burchakli uchburchakning s gipotenuzasi va a kateti berilgan. Ikkinchi katetni toping:

1)c= 5; $a=3$; 2) $c=13$; $a= 5$; 3) $c= 6$; $a= 5$.

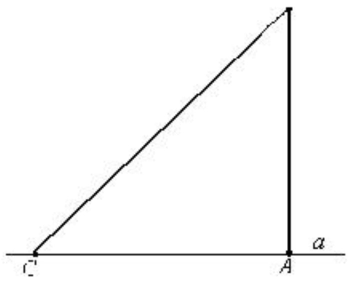
(*Javob*: 1) 4; 2) 12; 3) $\sqrt{11} \approx 3{,}3$).

3. To`g`ri burchakli uchburchakning ikki tomoni 3 m va 4 m ga teng. Uchinchi tomonni toping (ikkita holni ko`ring).

(*Javob*: 5 m yoki $\sqrt{7} \approx 2{,}6$ m).

Perpendikulyar va og`ma

BA kesma *a* To`g`ri chiziqqa *B* nuqtadan tushirilgan perpendikulyar va *C* nuqta *a* To`g`ri chiziqning *A* dan boshqa ixtiyoriy nuqtasi bo`lsin. *BC* kesma *B* nuqtadan *a* To`g`ri chiziqqa o`tkazilgan *oqma* deyiladi (2- rasm).



2-rasm.

C nuqta oqmaning *asosi* deyiladi. *AC* kesma oqmaning *a to*`g`ri chiziqdagi *proyeksiyasi* (soyasi) deyiladi.

Pifagor teoremasidan quyidagi xulosalar kelib chiqadi.

Agar bir nuqtadan To`g`ri chiziqqa perpendikulyar va oqmalar o`tkazilsa, istalgan oqma perpendikulyardan katta, teng oqmalar teng proyeksiyalarga ega, ikkita oqmadan qaysi birining proyeksiyasi katta bo`lsa, o`sha oqma katta bo`ladi.

ISSN: 2582-4686 SJIF 2021-3.261,SJIF 2022-2.889, 2024-6.875 ResearchBib IF: 10.25 / 2025

THE MULTIDISCIPLINARY JOURNAL OF SCIENCE AND TECHNOLOGY

VOLUME-6, ISSUE-3

Pifagor teoremasiga ko`ra: $AB^2 ++ AC^2 = BC^2$.

Bundan $BC > AB$ ekani ko`rinib turibdi. $AB$ berilgan $AC$ dan qancha katta bo`lsa, BC shuncha katta bo`ladi.

Mashqlar

1.Uchburchakning tomonlarida olingan ixtiyoriy ikki nuqta orasidagi masofa uning eng katta tomonidan katta emasligini isbotlang.

2.To`rtburchak diagonallarining kesishishi ma'lum. Ular uzunliklarining yiqindisi to`rtburchakning perimetridan kichik, ammo yarim perimetridan katta. SHuni isbotlang.

Uchburchaklardagi metrik munosabatlar

Geometrik shakllar ichida eng ko`p uchraydigan va geometrik masalalar yechishda ko`p qo`llaniladigan shakl bu uchburchakdir. SHuning uchun uchburchakka doir yoki uchburchak elementlarining kombinatsiyasi bilan yechiladigan masalalar ko`p uchraydi.

Uchburchak elementlarining kombinatsiyasi orqali beriladigan masalalar asosan quyidagi ko`rinishda bo`ladi:

1. Uchburchakning uchta tomoniga ko`ra beriladigan masalalar.

2. Uchburchakning ikki tomoni va ular orasidagi burchagiga ko`ra beriladigan masalalar.

3. Uchburchakning bir tomoni va unga yopishgan ikki burchagiga ko`ra beriladigan masalalar.

4. Uchburchakning ikki tomoni va bu tomonlardan biri qarshisidagi burchakka ko`ra beriladigan masalalar.

5. Uchburchakning bir tomoni hamda unga qarshi yotgan va yopishgan burchagiga ko`ra beriladigan masalalar.

Yuqoridagilarga qo`shimcha yana quyidagilarni yozish mumkin:

1. Uchburchakka tashqi chizilgan aylananing radiusi:

$$R = \frac{abc}{4S}.$$

2. Uchburchakka ichki chizilgan aylananing radiusi:

$$r = \frac{2S}{p}, \quad \text{bu yerda:} \quad p = \frac{a+b+c}{2}.$$

3. Uchburchakning balandliklari mos ravishda *ha*, *hb*, *hc* va ichki chizilgan aylananing radiusi *r* bo`lsa,

$$\frac{1}{r} = \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c}$$

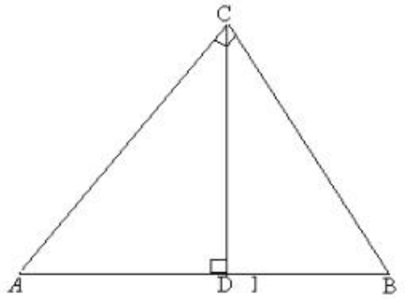
munosabat o`rinli bo`ladi.

ISSN: 2582-4686 SJIF 2021-3.261,SJIF 2022-2.889, 2024-6.875 ResearchBib IF: 10.25 / 2025

THE MULTIDISCIPLINARY JOURNAL OF SCIENCE AND TECHNOLOGY

VOLUME-6, ISSUE-3

4. To`g`ri burchakli uchburchakning To`g`ri burchagi uchidan uning gipotenuzasiga tushirilgan perpendikulyar gipotenuza bo`laklari orasida o`rta proportsional miqdordir. o`ar  bir katet butun gipotenuza bilan uning gipotenuzadagi proyeksiyasi orasida o`rta proportsional miqdordir, ya'ni (3-rasm):



3-rasm.

$AC^2=AB \cdot AD$,

$CD^2=AD \cdot BD$,

$BC^2=AB \cdot BD$.

5. Bu yuqoridagi munosabatlardan bevosita To`g`ri burchakli uchburchakning tomonlari bir xil o`lchovli bo`lganda katetlar kvadratlarining yiqindisi gipotenuzasining kvadratiga teng degan mulohazani isbotlash osondir, ya'ni (4- rasm):

$$AC^2+BC^2 = AB\bullet AD+AB\bullet BD = AB(AD+BD) = AB\bullet AB = AB^2 \Rightarrow$$

$$\Rightarrow AC^2+BC^2 = AB^2.$$

6. Uchburchak burchagining bissektrisasi uning shu burchak qarshisida yotgan tomonini qolgan tomonlariga proportsional burchaklarga bo`ladi (4- rasm), ya'ni

$$BD : DC = AB : AC,$$

bunda $AD = la$ – bissektrisa.

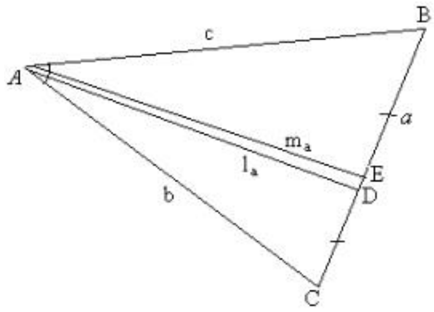
7. Uchburchak medianasi o`zi chiqqan burchak qarshisida yotgan tomonni teng ikkiga bo`ladi. Medianalarning uzunliklari ushbu formulalar bilan topiladi (5- rasm):

$$m^2{}_a = \frac{b^2+c^2}{2} - \frac{a^2}{4}, m^2{}_b = \frac{a^2+c^2}{2} - \frac{b^2}{4}, m^2{}_c = \frac{a^2+b^2}{2} - \frac{c^2}{4},$$

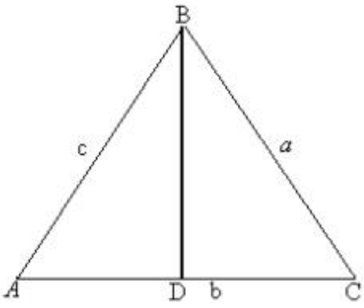
ISSN: 2582-4686 SJIF 2021-3.261,SJIF 2022-2.889, 2024-6.875 ResearchBib IF: 10.25 / 2025

THE MULTIDISCIPLINARY JOURNAL OF SCIENCE AND TECHNOLOGY

VOLUME-6, ISSUE-3



4-rasm.



5-rasm.

8. Agar berilgan ixtiyoriy uchburchakning tomonlari mos ravishda *a*, *b*, *c* bo`lsa, c tomonning *b* tomondagi proyeksiyasining uzunligi (58- rasm):

$$AD = \frac{c^2 + b^2 - a^2}{2b}$$

formula bilan topiladi.

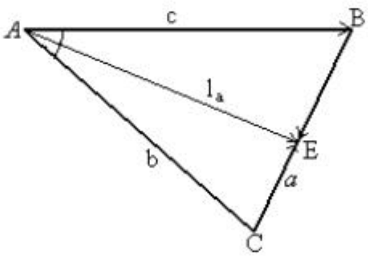
Masalalar ko`rib chiqamiz.

1-masala. *ABC* uchburchakning tomonlari *a*, *b*, *c* ga teng. SHu uchburchakning *a* tomoniga o`tkazilgan *la* bissektrisaning uzunligini hisoblang.

Berilgan: ΔABC; $AB = c$; $AC = b$; $BC = a$ (6- rasm).

Topish kerak: $AE = la = ?$

Echilishi. Uchburchak bissektrisasining xossasiga asosan $AB : AC = BE : EC$ ni yoza olamiz. Agar uchburchak tomonlarini vektorlar orqali ifodalasak, u holda *AE* bissektrisaning



6-rasm.

$$\overline{AE} = \frac{\overline{CE} \cdot \overline{AB} + \overline{BE} \cdot \overline{AC}}{\overline{CE} + \overline{BE}}$$

vektorli ifodasini yozish mumkin. Bu ifodaning ikkala tomonini kvadratga oshirsak,

$$\overline{AE}^2=\frac{\overline{CE}^2\cdot\overline{AB}^2+\overline{BE}^2\cdot\overline{AC}^2+2\overline{CE}\cdot\overline{AB}\cdot\overline{BE}\cdot\overline{AC}}{\overline{CE}^2+\overline{BE}^2+2\overline{CE}\cdot\overline{BE}}$$

vektorli ifodani hosil qilamiz. 59- rasmda $\overline{BC}=\overline{AC}-\overline{AB}$ ekanini hisobga olib, bu tenglikning ikkala tomonini kvadratga oshirsak,

$$\overline{BC}^2=\overline{AC}^2+\overline{AB}^2-2\overline{AC}\cdot\overline{AB}$$

vektorli tenglikka ega bo`lamiz, bundan

$$2\overline{AC}\cdot\overline{AB}=\overline{AC}^2+\overline{AB}^2-\overline{BC}^2$$

ekanini hisobga olsak, u holda

$$\overline{AE}^2=\frac{\overline{CE}^2\cdot\overline{AB}^2+\overline{BE}^2\cdot\overline{AC}^2+\overline{CE}\cdot\overline{BE}\cdot(\overline{AC}^2+\overline{AB}^2-\overline{BC}^2)}{\overline{CE}^2+\overline{BE}^2+2\overline{CE}\cdot\overline{BE}}$$

tenglikni hosil qilamiz. O`ng tomondagi kasrning surat va maxrajini *BE•CE* ga bo`lsak,

$$\overline{AE}^2=\frac{\frac{CE}{BE}\overline{AB}^2+\frac{BE}{CE}\cdot\overline{AC}^2+\overline{AC}^2+\overline{AB}^2-\overline{BC}^2}{\frac{CE}{BE}+\frac{BE}{CE}+2}=\frac{\frac{b}{c}c^2+\frac{c}{b}\cdot b^2+b^2+c^2-a^2}{\frac{b}{c}+\frac{c}{b}+2}=\frac{bc}{(b+c)^2}\cdot 4p(p-a)$$

hosil bo`ladi, bu yerda . $p=\dfrac{a+b+c}{2}$

Demak,

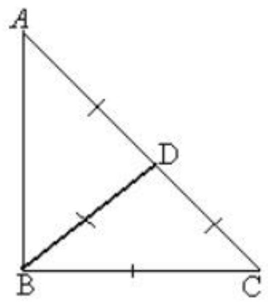
$$l_a=\frac{2}{b+c}\sqrt{bcp(p-a)},$$

shunga o`xshash uchburchakning *b* va *c* tomonlariga o`tkazilgan bissektrisalar uzunligi uchun

$$l_b=\frac{2}{a+c}\sqrt{acp(p-b)}\quad \text{va}\quad l_c=\frac{2}{a+b}\sqrt{abp(p-c)}$$

formulalarni hosil qilish mumkin.

2-masala. Agar teng yonli uchburchakning asosidagi burchakning biridan chiqqan To`g`ri chiziq uni ikkita teng yonli uchburchakka ajratsa, berilgan teng yonli uchburchakning burchaklarini toping (7-rasm).



Echilishi. *ABC* uchburchakda *AB* =*AC* va *D* nuqta *AC* tomonda yotib, *ABC* uchburchakni Δ *ADB* va Δ *DBC* larga ajratadi, bunda *AD* =*BD* =*BC*.

Agar $\angle$ *ABD* = *x* deb olsak, $\angle$ *BCD*= $\angle$ *BDC*= 2*x* bo`ladi. *AB* =*AC* bo`lganligidan $\angle$ *CBD* =*x* bo`ladi. Bundan 5*x* =$180^0$ hosil bo`lib, *x*=36^0 ekani kelib chiqadi.

ISSN: 2582-4686 SJIF 2021-3.261,SJIF 2022-2.889, 2024-6.875 ResearchBib IF: 10.25 / 2025

Foydalanilgan adabiyotlar.

1. Isroilov I., Pashayev Z. GEOMETRYIA. 1-qism. Akademik litseylar uchun darslik. 2-nashri. "O ‘qituvchi", 2010. 224 betlik.

2. Этан Уотрелл, Норберт Гербер. Эффективная работа: Flash MX. Перевод с английского языка В. Кочерги. Москва-Санкт-Петербург. 2003. 360 с.