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A STATICALLY INDETERMINATE PROBLEM UNDER TENSION AND COMPRESSION

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Introduction. A rod with a stepped non-uniform cross-section, rigidly fixed at both ends, is subjected to concentrated axial loads (Fig.1). It is required to construct the diagrams of axial forces, normal stresses, and axial displacements. In the calculations, assume $E = 2 \cdot 10^5$ MPa $= 2 \cdot 10^6$ kg/sm^2.

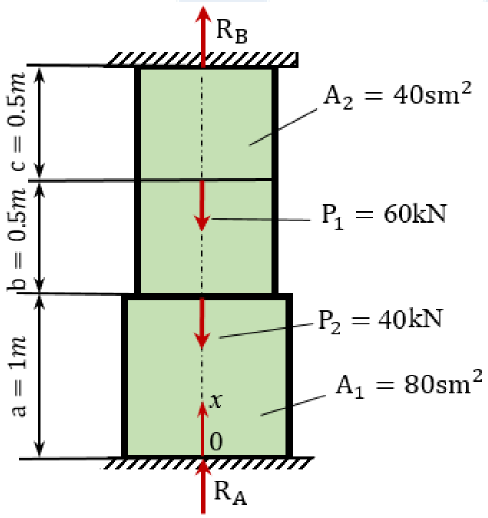

Fig. 1. Rod diagram

Let us project the loads applied to the rod and the support reactions onto its axis:

$$\sum F_{kx} = 0, \qquad R_A + R_B - 40 - 60 = 0$$

$$R_A + R_B = 100$$

This equation is insufficient to determine the support reactions. The rod is statically indeterminate of the first degree; therefore, an additional equation must be formulated for its analysis, based on the character of the rod's deformation. Let us remove one of the fixed supports of the rod, for example the upper one, and replace its effect by the unknown support reaction.
$x = R_B$ (Fig.1).

The statically determinate system thus obtained is called the primary (basic) system.

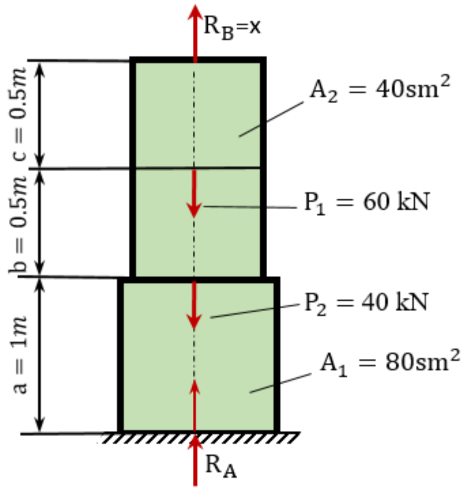

Fig. 2. Primary system

Let us impose the condition that the deformations of the primary system and the given system are identical. The length of a rod rigidly fixed at both ends cannot change after the application of loads ($\Delta l = 0$). Therefore, for the primary system, on the basis of the principle of superposition (independence of the action of forces), one can write:

$$\Delta l = \Delta l_p + \Delta l_x = 0$$

Where Δl_p – the deformation of the primary system under the action of the given loads,

Δl_x – the deformation of the primary system under the action of the unknown support reaction $x = R_B$.

We calculate the values of Δl_p и Δl_x and formulate the additional equation:

$$\Delta l_p = -\frac{6000 \cdot 50}{2 \cdot 10^6 \cdot 40} - \frac{6000 \cdot 100}{2 \cdot 10^6 \cdot 80} - \frac{4000 \cdot 100}{2 \cdot 10^6 \cdot 80} = -0{,}01 \text{ sm}$$

$$\Delta l_x = \frac{x \cdot 100}{2 \cdot 10^6 \cdot 40} + \frac{x \cdot 100}{2 \cdot 10^6 \cdot 80} = 1{,}875 \cdot 10^{-6} \cdot x$$

$$\Delta l = \Delta l_p + \Delta l_x = -0{,}01 + 1{,}875 \cdot 10^{-6} \cdot x = 0$$

$$x = R_B = 5333 \text{ kg} = 53{,}3 \text{ kN}$$

The static indeterminacy of the problem has been resolved. We calculate the axial forces and normal stresses in the characteristic sections of the rod, starting from the free end of the primary system.

1. Section: $x = 2$ m

$N = x = R_B = 53{,}3$ kN (Tension),

$$\sigma = \frac{5333}{40} = 133 \text{ kg/sm}^2 = 13{,}3 \text{ MPa}$$

2. Section: $x = 1{,}5 + 0$

$N = 53{,}3$ kN, $\sigma = 13{,}3$ MPa

3. Section: $x = 1{,}5 - 0$

$N = 53{,}3 - 60 = -6{,}7$ kN (Compression)

$$\sigma = \frac{-670}{40} = -17 \text{ kg/sm}^2 = -1{,}7 \text{ MPa}$$

4. Section: $x = 1 + 0$

$N = -6{,}7$ kN, $\sigma = -1{,}7$ MPa

5. Section: $x = 1 - 0$

$N = -6{,}7 - 40 = -46{,}7$ kN (Compression)

$$\sigma = \frac{-4670}{80} = -58 \text{ kg/sm}^2 = -5{,}8 \text{ MPa}$$

6. Section: $x = 0$
$N = -46{,}7$ kN, $\sigma = -5{,}8$ MPa

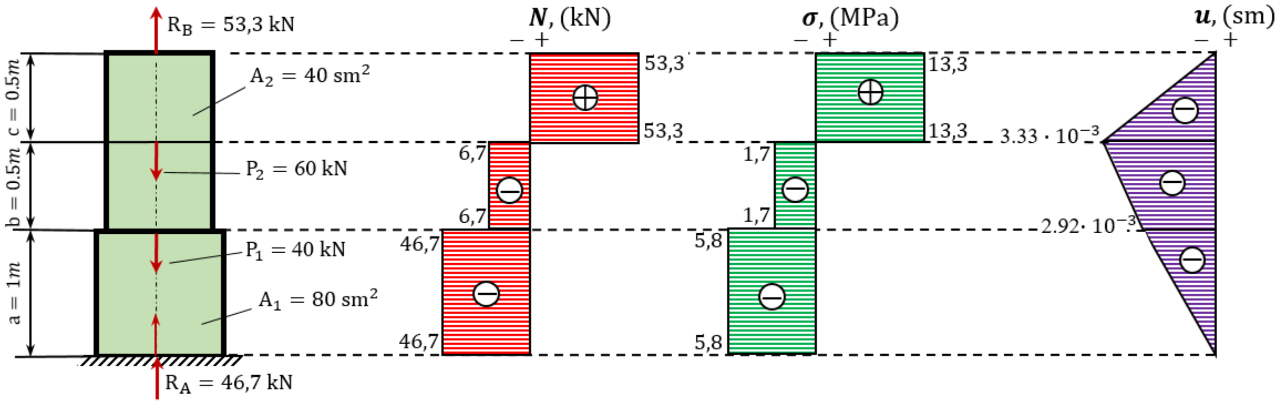

Fig. 1.2. Diagrams N, σ and u

Based on the calculated values, we construct the diagrams of N and σ. Note the presence of jumps in the N diagram at the sections where concentrated forces are applied. Within each segment, N and σ remain constant in magnitude.

We calculate the absolute deformations of the segments of the rod:

$$\Delta l_1 = \frac{-4670 \cdot 100}{2 \cdot 10^6} = -2{,}92 \cdot 10^{-3} \text{ sm (shortening)}$$

$$\Delta l_2 = \frac{-670 \cdot 50}{2 \cdot 10^6} = -4{,}1 \cdot 10^{-4} \text{ sm (contraction)}$$

$$\Delta l_3 = \frac{5333 \cdot 50}{2 \cdot 10^6} = 3{,}33 \cdot 10^{-3} \text{ sm (elongation)}$$

The absolute deformation of the entire rod must be equal to zero. Let us verify the fulfillment of this condition:

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3 = -2{,}92 \cdot 10^{-3} - 4{,}1 \cdot 10^{-4} + 3{,}33 \cdot 10^{-3} = 0$$

We calculate the axial displacements of the characteristic sections of the rod, starting from the lower fixed section:

1. Section: $x = 0$ $u = u_0 = 0$
2. Section: $x = 1$ m $u_1 = u_0 + \Delta l_1 = -2{,}92 \cdot 10^{-3}$ sm
3. Section: $x = 1{,}5$ m $u_2 = u_1 + \Delta l_2 = -2{,}92 \cdot 10^{-3} - 4{,}1 \cdot 10^{-4} = -3{,}33 \cdot 10^{-3}$ sm
4. Section: $x = 2$ m $u_3 = u_2 + \Delta l_3 = \Delta l = 0$

The diagram of axial displacements is shown in Fig. 1.2. The axial displacements vary linearly. All sections of the rod move in the negative *x*-axis direction, i.e., downward.

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